NUMERICAL CALCULATION OF THE ATTENUATION PRODUCED BY REACTIVE SILENCERS

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ABSTRACT

This work shows the application of a finite difference algorithm to simulate the unsteady compressible flow in reactive silencers in order to obtain their sound attenuation. The numerical method solves the conservation equations of mass, momentum and energy. The method is applied to four different configurations of silencers including the expansion chamber, the Helmholtz resonator, the quarter-wave resonator and the Quincke tube. Comparison for the transmission loss between the theoretical and the calculated values are in good agreement for these geometries.

RESUMEN

En este trabajo se presenta la aplicación de un programa de cálculo basado en una discretización en diferencias finitas unidimensional para la simulación del flujo compresible en silenciadores de tipo reactivo, con el objeto de calcular su atenuación. El algoritmo desarrollado resuelve las ecuaciones no estacionarias de conservación de masa, de cantidad de movimiento y de energía en el conducto, para obtener la evolución temporal y espacial de las características del fluido, considerando que por fuente se tiene un pistón oscilante. La solución temporal se convierte al dominio frecuencial utilizando la transformada rápida de Fourier, a partir de la cual se puede obtener la atenuación producida por el silenciador.

Para mostrar la bondad del algoritmo desarrollado, se presenta la comparación de resultados numéricos con resultados experimentales para diversas geometrías de silenciador, como la expansión brusca, el resonador de Helmholtz, el resonador lateral y el tubo de Quincke, utilizando para todos ellos tanto una terminación ecoica como anecoica.

INTRODUCTION

Basically there are two main groups of silencers: dissipative and reactive. Dissipative silencers attenuate sound energy incorporating an absorbent material that is usually fibrous or porous, while reactive silencers reduce the noise by means of a change in the transversal section of the duct, inducing a wave reflection. The latter ones are found in the exhaust pipe of internal combustion engines, but also in ventilation and air conditioning ducts. The geometries for reactive silencers are the expansion chamber, the Helmholtz, the quarter-wave resonator and the Quincke tube.

In a previous work (Ballesteros et al., 1998) an study of the behaviour of reactive silencers was presented. In that work, three geometries (the expansion chamber, the Helmholtz resonator and the quarter-wave resonator) with anechoic termination were studied. In this work, we present the extension of the model to a new geometry (the Quincke tube) and to all the geometries with echoic termination.

GOVERNING EQUATIONS AND SILENCERS DISCRETIZATION

The objective of this study is to simulate the one-dimensional compressible flow in a reactive silencer with a finite difference technique. The numerical method, coherent with the work of Chapman, Novak and Stein (1982), is applied to the conservation equations of mass, momentum and energy, assuming a perfect gas law. For a compressible unsteady flow, these equations can be written as follows:

-Conservation of mass: $$\frac{\partial \rho}{\partial t}+\nabla(\rho \bar{U})=0$$

-Conservation of momentum: $$\frac{\partial}{\partial t}(\rho \bar{U})+\nabla(\rho \bar{U}\bar{U})+\nabla p-\nabla\bar{\tau}=0$$

-Conservation of internal energy: $$\frac{\partial}{\partial t}(\rho e)+\nabla(\rho \bar{U} e)+p\nabla \bar{U}-\bar{\tau}\div\nabla \bar{U}+\nabla \bar{q}=0$$

with the perfect gas law: $$p=(\gamma-1)\,\rho\, e$$

These equations are discretized in ducts of variable cross section using an explicit finite difference algorithm, obtaining the following expressions:

-Equation for the cell density: $$\frac{\rho^{n+1}_{j+\frac{1}{2}}-\rho^{n}_{j+\frac{1}{2}}}{\Delta t}+\left(\frac{M^{n+\frac{1}{2}}_{j+1}-M^{n+\frac{1}{2}}_{j}}{V_{j+\frac{1}{2}}}\right)=0$$

where the superscript represents the temporal index of an specific variable.

-Equation for the velocity on the cell face:

$$\frac{M_j^{n+1}U_j^{n+1}-M_j^{n}U_j^{n}}{\Delta t}+U^{n}_{j+\frac{1}{2}}M^{n+\frac{1}{2}}_{j+\frac{1}{2}}-U^{n}_{j-\frac{1}{2}}M^{n+\frac{1}{2}}_{j-\frac{1}{2}}+\left(p^{n+\frac{1}{2}}_{j+\frac{1}{2}}-p^{n+\frac{1}{2}}_{j-\frac{1}{2}}\right)a_j+\tau^{n}_{w,j}a_{w,j}=0$$

where M^n_j is the cell mass, $U^n_{j+1/2}$ is the cell velocity, $M^{n+1/2}_{j+1/2}$ is the cell mass flow, $p^{n+1/2}_{j+1/2}$ is the cell pressure, $\tau^n_{w,j}$ is the shear stress and $a_{w,j}$ is the cell area.

-Equation for the internal energy of the cell:

$$\frac{M^{n+1}_{j+\frac{1}{2}}e^{n+1}_{j+\frac{1}{2}}-M^{n}_{j+\frac{1}{2}}e^{n}_{j+\frac{1}{2}}}{\Delta t}+\left(M^{n+\frac{1}{2}}_{j+1}e^{n+\frac{1}{2}}_{j+1}-M^{n+\frac{1}{2}}_{j}e^{n+\frac{1}{2}}_{j}\right)+p^{n+\frac{1}{2}}_{j+\frac{1}{2}}\left(U^{n+\frac{1}{2}}_{j+1}a_{j+1}-U^{n+\frac{1}{2}}_{j}a_{j}\right)-$$

$$-\tau^{n}_{w,j+\frac{1}{2}}a_{w,j+\frac{1}{2}}U^{n+\frac{1}{2}}_{j+\frac{1}{2}}+h_{T,j+\frac{1}{2}}a_{w,j+\frac{1}{2}}\left(T^{n}_{j+\frac{1}{2}}-T_{\infty}\right)=0$$

where h_T is the total heat transfer coefficient.

-The cell pressure is calculated using the perfect gas law: $$p^{n+1}_{j+\frac{1}{2}}=(\gamma-1)\,\rho^{n+1}_{j+\frac{1}{2}}e^{n+1}_{j+\frac{1}{2}}$$

These equations are applied to all the cells in order to obtain the variables for the time step n+1; then variables at the time step n are replaced by the variables at n+1 and the process is repeated until convergence is reached.

The staggered mesh used divides a duct into cells of the same length. While vector quantities are located at the cell faces, (index j), scalar magnitudes are placed at the cell centre (index j±1/2) (Figure 1).

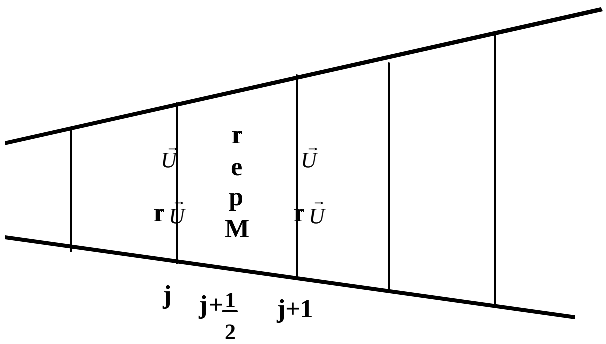


Figure 1

The noise source is modelled as an oscillating piston with a velocity given by $U_p^n = U_O \cos(wt)$ where ω is the frequency and U_O is the maximum velocity.

As the algorithm is explicit, the time increment is calculated with the Courant condition: $\Delta t < \frac{\Delta x}{c + |U|}$ where c is the velocity of sound, Δx is the cell length and U is the velocity of the cell faces. This increment of time is reduced by a safety factor to $\Delta t = 0.5 \frac{\Delta x}{c + |U|}$.

The expansion chamber is modelled as a duct and divided in cells of constant length Δx and variable cross section. At both ends the cross-section is smaller compared with the silencer cross section (Figure 2). The Helmholtz (Figure 3) and the quarter-wave (Figure 4) resonators are modelled as two ducts connected in a junction and divided in cells of equal length Δx and variable cross section. Figure 5 shows the geometry of the Quincke tube.

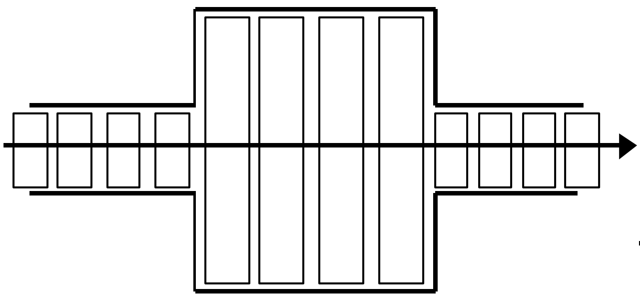
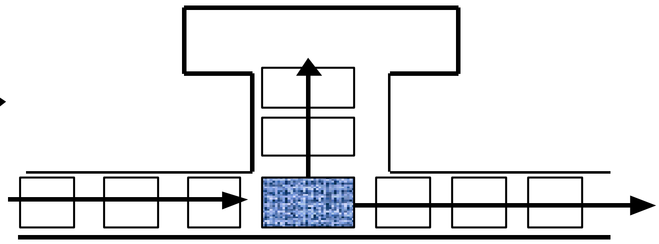


Figure 2



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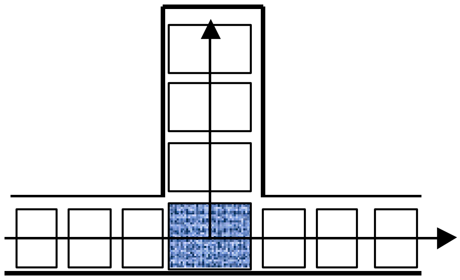


Figure 4

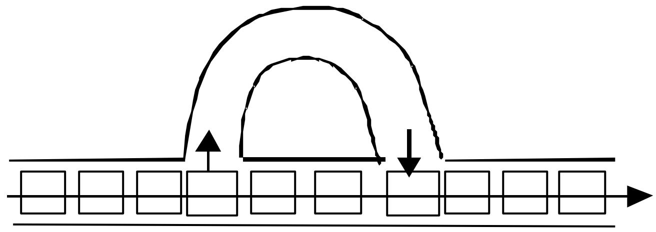


Figure 5

After the silencer two possibilities were simulated:

1) Anechoic termination: this termination dissipates waves that are transmitted through the silencer without reflection. This is convenient for analytical studies since the absence of wave reflection at the exit causes the transmission loss to be determined solely by the acoustic properties of the silencer.
2) Echoic termination: in practice it is rare to encounter a silencer with a non-reflecting termination; the mos likely case includes termination by an exit duct of finite length, where the waves propagating to the atmospheric interface are reflected back toward the source. These tailpipe reflections have an effect on the transmission loss characteristics of a silencer and may even cause an increase of the original signal.

CALCULATION OF THE ATTENUATION

It can be shown that the solutions of the wave equation for the pressure fluctuation can be represented as: $p(x,t) = \mathrm{Re}\{p(x)\,e^{jwt}\} = \mathrm{Re}\{(A\,e^{-jkx} + B\,e^{jkx})e^{jwt}\}$, where A is the positive wave amplitude and B is the negative wave amplitude. The attenuation is calculated in terms of the transmission loss, defined as the ratio between the sound power entering and leaving the silencer. If the inlet and outlet ducts have the same cross-sectional area, as it is the case in this study, the ratio of sound powers becomes equal to the ratio of the maximum sound pressure amplitudes. Then, the transmission loss (in dB) is given by

$$\mathrm{TL}(w) = 10 \log_{10} \left| \frac{C_{+,i}(w)}{C_{+,t}(w)} \right|^2$$

where $C_+(\omega)$ is the magnitude of the fluctuating pressure in the positive direction at a frequency ω, and the subscripts i and t refer to incident and transmitted components, respectively. Once the pressure fluctuations before and after the silencer are obtained in the time domain, the positive wave has to be calculated by means of the following expression:

$$C_+(w) = p(x,t) \frac{\mathrm{tg}(w\,t + k\,x)}{\cos(w\,t - k\,x)\,\mathrm{tg}(w\,t + k\,x) - \mathrm{sen}(w\,t - k\,x)}$$

which is obtained from the wave equation. The Fast Fourier Transform is applied to these signals in order to obtain the transmission loss at the required frequency.

RESULTS

The described algorithm was applied to several configurations of the four silencers described before with both types of termination. A summary of the results is presented here.

a) Simulation with echoic termination. Figures 6 and 7 show the time evolution of the pressure and the velocity at the first and at the last cell for a Helmholtz resonator at a frequency of 50 Hz. Figure 8 shows the calculated attenuation compared with the transmission loss predicted with the acoustic theory applied to this geometry. The calculation process for each frequency requires several time steps until convergence is reached. For this case the lengths before and after the silencer were 0.2 m and 1.5 m respectively, and the volume of the resonator was $9{\cdot}10^{-3}$ m^3.

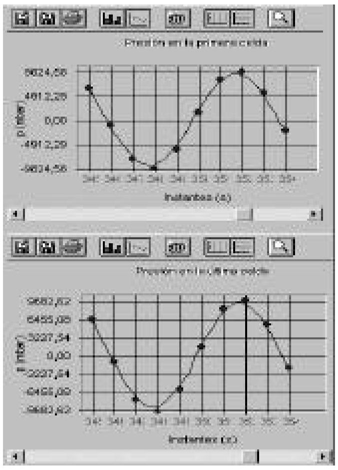
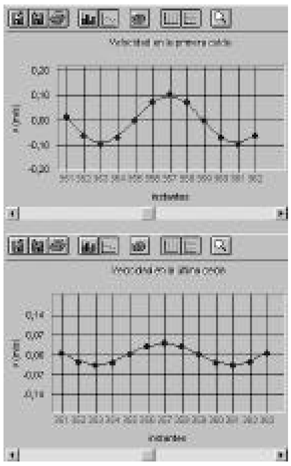


Figure 6



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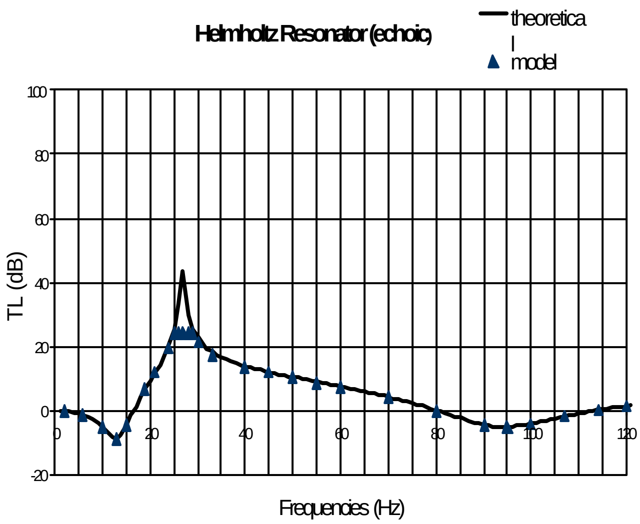

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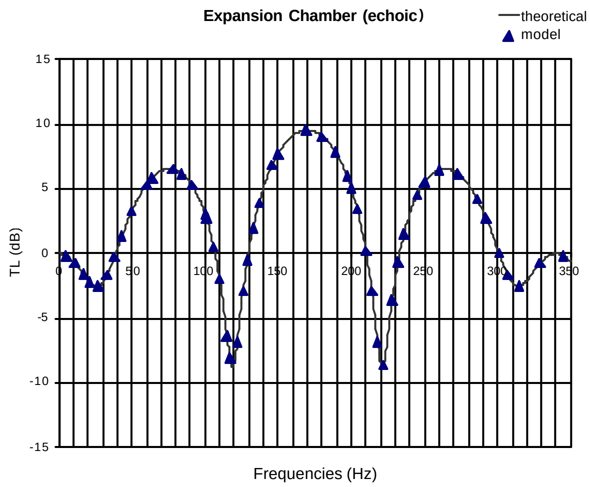

Figure 9

Figures 9 to11 show the comparison between the calculated attenuation and the transmission loss predicted with the acoustic theory for the expansion chamber, the quarter wave resonator an the Quincke tube. In all cases, good agreement is found between both sets of data throughout the frequency range, except in the resonance region of the Helmholtz resonator, probably due to the three-dimensional effects, not included in the model.

b) Simulation with anechoic termination. Figures 12 and 13 show the time evolution of the pressure and the velocity at the first and at the last cell for an expansion chamber resonator at a frequency of 50 Hz. Figure 8 shows the calculated attenuation compared with the transmission loss predicted with the acoustic theory applied to this geometry. The calculation process for each frequency requires several time steps until convergence is reached. For this case the lengths before and after the silencer were 0.2 m and 1.5 m respectively, and the volume of the resonator was $9 \cdot 10^{-3}$ m^3.

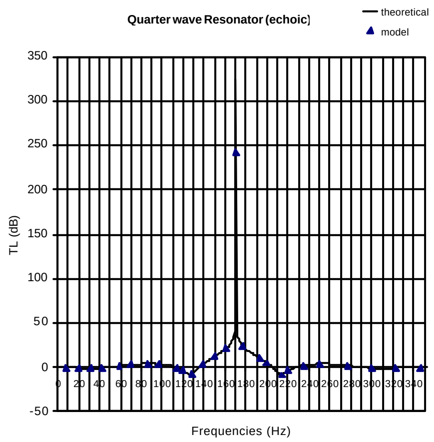

Figure 10

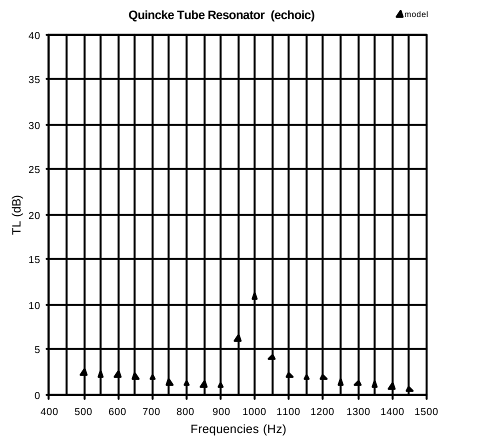

Figure 11

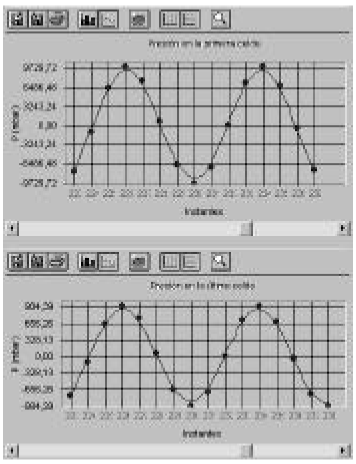

Figure 12

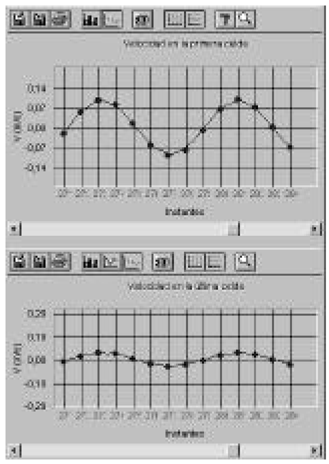

Figure 13

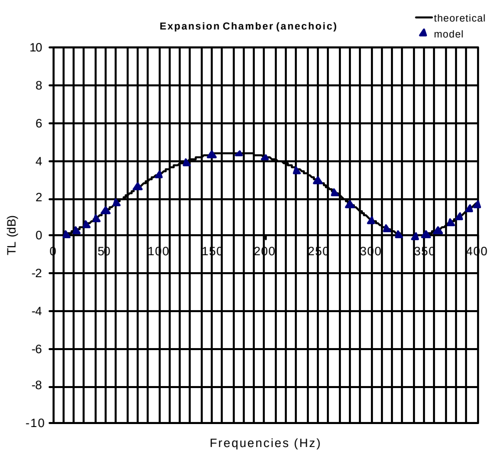

Figure 14

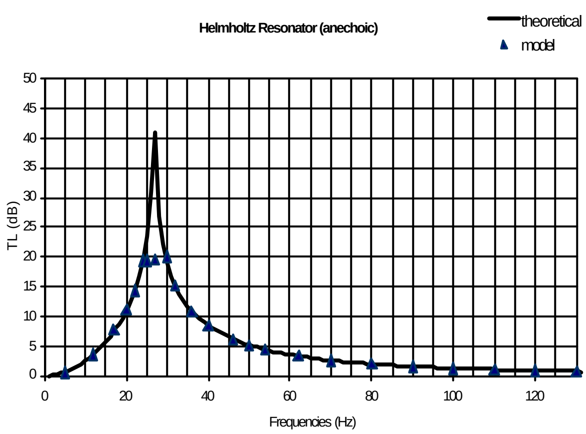

Figure 15

Figures 9 to11 show the comparison between the calculated attenuation and the transmission loss predicted with the acoustic theory for the expansion chamber, the quarter wave resonator an the Quincke tube. In all cases, good agreement is found between both sets of data throughout the frequency range, except in the resonance region of the Helmholtz resonator, probably due to the three-dimensional effects, not included in the model.

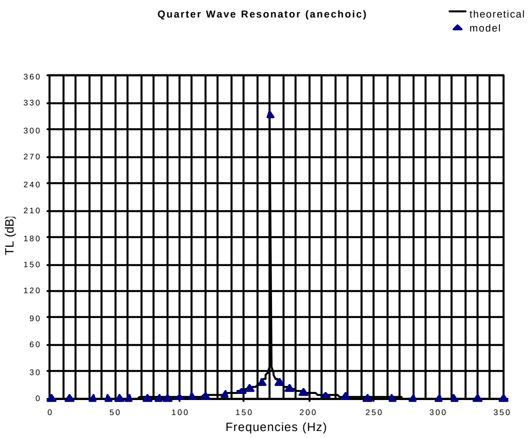

Figure 16

Quincke Tube (anechoic)

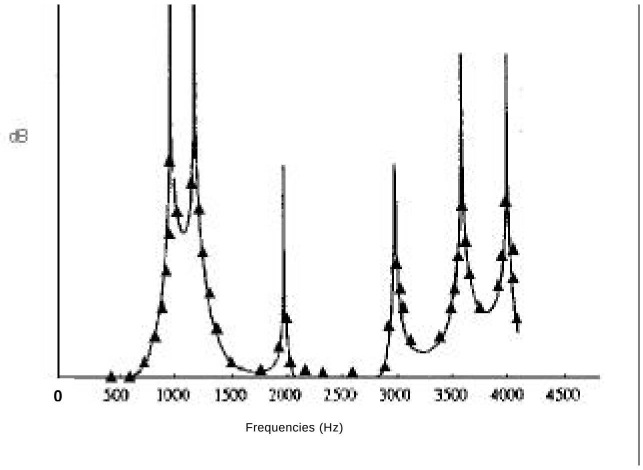

Figure 17

CONCLUSIONS

A one-dimensional finite difference algorithm of the conservation equations for unsteady compressible flow has been developed to study the attenuation of sound in reactive silencers with finite-tailpipe termination. The algorithm was applied to different configurations of reactive silencers and the results showed a remarkable agreement with the theoretical transmission loss predicted with the acoustic theory. This model can also be extended to a wide range of applications, such as complex geometries, non-linear flow phenomena and entire engine intake and exhaust systems.

ACKNOWLEDGMENTS

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