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NONLINEAR SOUND REVERBERATION IN ENCLOSURES

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Abstract
A model on nonlinear sound reverberation of enclosures will be presented. The classical statistical theory of sound propagation in enclosures has been enlarged to include non linear excitation signals. Numeric computations as well as experimental results indicate reverberation times shorter than under linear conditions. An equation relating reverberation time under nonlinear conditions with reverberation time under linear conditions through an intensity nonlinear parameter has been derived. Some repercussions on experimental room acoustic studies (actual and reduced scale room models) will also be commented.

Keywords: sound reverberation, nonlinear signals, scale room models.

1 Introduction

This document defines the template to be used in manuscript preparation for the Congress Acústica 2008.

2 Sound reverberation in the linear range

The statistical model of sound propagation in enclosures developed first by Jaeger [1] gave a solid support to the model empirically developed some years before by Sabine [2]. A different approach, under similar assumptions, was offered by Eyring [3] only some years later obtaining a different equation that distinguish among different sound absorption coefficients of different surfaces bounding the enclosure. An equivalent but much more simple and elegant development was given by Norris [4].

Once a sound field under diffuse and homogeneous conditions in an enclosure attains steady conditions for intensity I_0, the acoustic power emitted by the source is cancelled by absorption in the boundaries and in the fluid itself. The absorption rate at the unit time within the fluid can be accounted by e^{-mc}, where m is the fluid absorption by unit length.

On an average along a mean free path ($\lambda_m = 4V/S$ for an enclosure of volume V and total surface S) the sound impacting on boundaries, is absorbed at a rate $(1-\bar{\alpha})^{cS/4V}$, where $\bar{\alpha} = \Sigma_i(S_i\alpha_i/S)$ is the mean absorption coefficient, and α_i is that one corresponding to surface S_i.

If the sound source is stopped at instant t = 0, the time evolution of intensity is given by

$$I = I_0[e^{-mct}\cdot\left(1-\bar{\alpha}\right)^{\frac{Sct}{4V}}] = I_0 e^{-[m-\frac{S\cdot\ln\left(1-\bar{\alpha}\right)}{4V}]ct} = I_0 e^{-[\frac{4mV-S\cdot\ln\left(1-\bar{\alpha}\right)}{4V}]ct} = I_0 e^{-\frac{t}{13,8\cdot T_r}} . \quad (1)$$

where the reverberation time is given by

$$T_r = \frac{55,3V}{c[4mV - S\ln\left(1-\bar{\alpha}\right)} . \quad (2)$$

In more or less different ways most numeric programs use these ideas as basic foundations.

From the previous equation it is to note that for a given enclosure, within the lineal range, the product $T_r\, c$ is an invariant. From the well known equation giving the speed of sound $c = \sqrt{(\gamma RT/M)}$, (γ being the ratio of the specific heat at constant pressure to that at constant volume, R the gas constant, T the absolute temperature and M the fluid's molecular weight of the fluid) it can be seen that temperature increases of 10 ºC cause increases of about 6 m/s in sound speed. On the other hand it can be found by using the invariance of Tc within the linear range that these changes lead to changes of only 2% in reverberation time (T c =(T+δ)(c+6) =Tc+6T +δ(c+6) → (δ /T) = -(6/(c+6)) = -0.02), a value within usual error ranges in measurements. Then significant increases of temperatures around reference atmospheric conditions cause quite limited changes in reverberation time.

3 Sound reverberation in the nonlinear range

3.1 General considerations

When dealing with nonlinear sound waves it is quite common to maintain the speed of sound as a constant [5], and later we will use this hypothesis, though at high nonlinear levels the sound speed is higher than within lineal range [6], [7], and causes among other effects distortion of nonlinear sound waves. It is also known since Sedov [7] that sound level decreases with distance to sound source at rates higher than within linear range. At very high nonlinear ranges it is admitted a level attenuation well above the linear range: $1/r^6$ for atomic explosions, for instance, compared to $1/r^2$ for linear range, due to simple spherical divergence.

In the following we will use $\beta=\kappa-2$, the difference between the nonlinear and linear exponents of r to account for attenuation with distance to source, as nonlinear indicator. Within the weak nonlinear range, (say below 150 dB), and for crack reports up to 2 g of powder, Moreno et al. [8], [9] have found these differences to be of the order of 0.5 for the complete signal, and up to 2.4 for 1/3 octave band centred around 1600 Hz [10], (the last at distances lower than 1 m).

3.2 Reverberation under nonlinear conditions

Concerning reverberation in enclosures, the main hypothesis of linear range holds. Additional conditions in the nonlinear range concern sound absorption, sound speed and excess attenuation with distance.

Therefore the following equation can be written, where subindex *a* has been used to label nonlinear quantities:

$$I_a = I_{0,a}[(c_a t)^{-\beta} e^{-m c_a t} \cdot \left(1-\bar{\alpha}\right)^{\frac{S c_a t}{4V}}] = I_{0,a} e^{-\beta \cdot \ln(c_a t)} e^{-[\frac{4mV - S \cdot \ln\left(1-\bar{\alpha}\right)}{4V}] c_a t} \quad . \tag{3}$$

Within the weak nonlinear range it has been shown [9], that for practical purposes *m* take the same values than within the linear range. By applying the definition of reverberation time to previous equation, it is found:

$$\frac{I_a}{I_{0,a}} = 10^{-6} = e^{-\beta \ln(c_a T_{ra})} e^{-[\frac{4mV - S \cdot \ln\left(1-\bar{\alpha}\right)}{4V}] c_a T_{ra}} \quad . \tag{4}$$

Applying logarithms and comparing with previous results for linear range, it is obtained after some simplifications the following equation:

$$\frac{c_a T_{ra}}{c T_r} = 1 - (\beta / 13.8) \cdot \ln(c_a T_{ra}) \quad . \tag{5}$$

This equation can be numerically solved. To that end practical intervals of cT_r, $c_a X_a$ and β, were chosen. Figure 1 gives a graphical representation of equation (5).

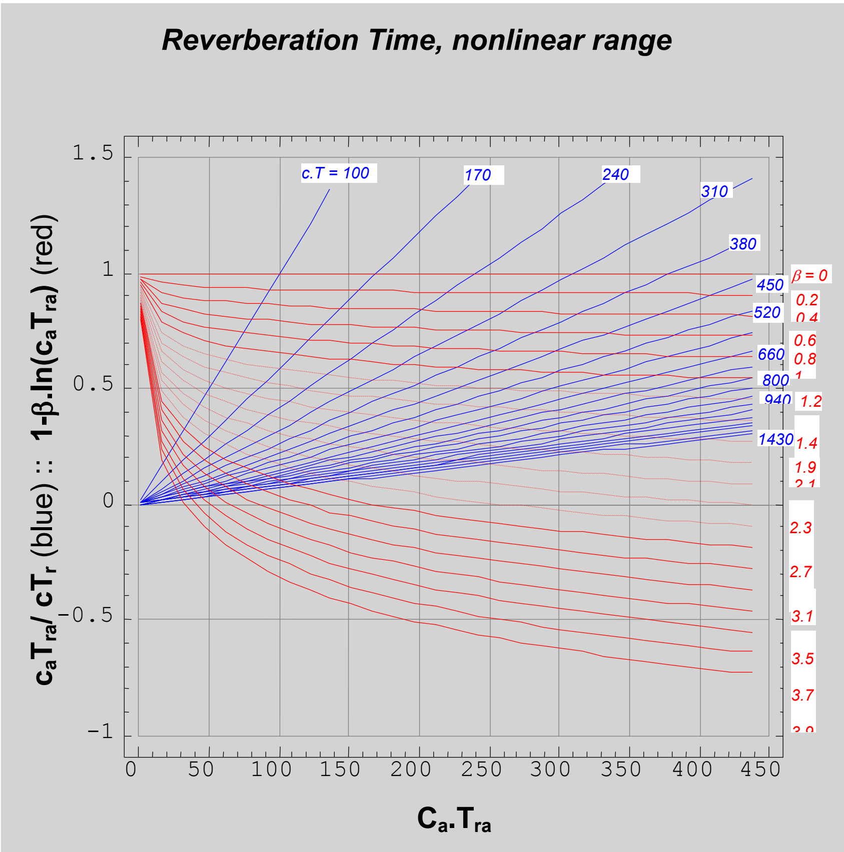

Figure 1 – Graphical representation of equation (5).

Solutions correspond to crossing points of red curves with blue curves where equation (10) holds. For instance for β=0, that corresponds to linear range, the set $c_a T_a$ =100 170 240 310 380, etc., is obtained and that set is exactly coincident to the set c.T =100 170 240 310 380, etc., of linear range.

According to that figure solutions are restricted to the interval $0 < (c_a T_a / cT) \leq 1$, and given $c \leq c_a$ it can be concluded that $T_a \leq T$. In words the reverberation time of any enclosure under nonlinear conditions is lower or equal to the reverberation time under linear conditions.
For values of the nonlinear parameter $\beta \geq 1.7$, it can be seen that $T_a \leq T/2$ and if reverberation equations are used to derive absorption (Sabine's equation for instance), abnormally high values will be found. This result is of particular importance in experimentation where 'nonlinear signals' (the above mentioned crack reports, for instance) be used.

Applying now the very likely assumption $c_a = c$, within the range of the weak nonlinearity, the set of 'point solutions' of Figure 1,numeric solutions leads to the Figure 2, that relates reverberation time under nonlinear conditions to reverberation time under linear conditions. The value $c=345$ *m/s*, corresponding to normal atmospheric conditions, has been used.

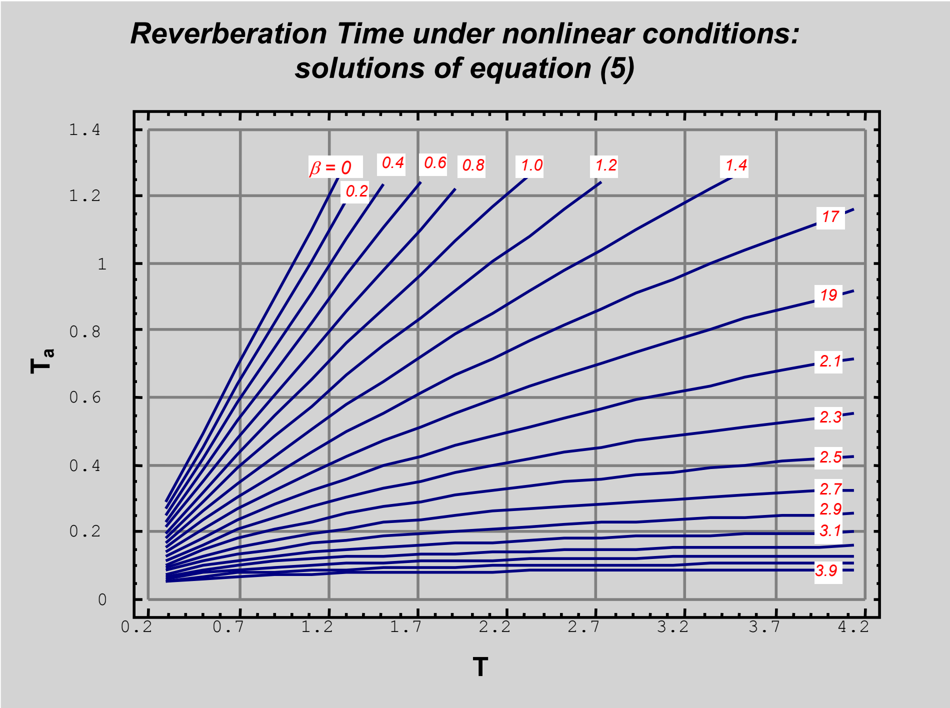

Figure 2 – Solutions $T_a = f(T,\beta)$ of equation (5).

Previous figure suggest the adequacy of fitting functions of the following type within the weak nonlinear range:

$$T_{a,j} = b_{1,j} \cdot T_1^{b_{2,j}} \quad . \tag{6}$$

where

$j=1,2\ 3,\ldots$, sweeps the set of curves for different values of β, and

T_1 , (instead of T) means that the value $c=345\ m/s$, has been used, as already indicated.

The set $b_{1,j}$ found numerically, can be expressed, by the least square method, in terms of polynomials of β. Degree 2 suffices to achieve good fittings. A similar procedure applies for $b_{2,j}$. The two vectors of coefficients found are respectively:
(-0.00111397 -0.409664 -0.0831248) and (0.998213 -0.0530357 -0.0513786).
For $\beta=0$, the condition $T_{r,a}=T_1$ holds and then the first coefficient of every vector should be 0 and 1 respectively. It is to note that computed values are quite good approaches. Rounding to these values and combining equations (6) it comes:

$$T_a = \frac{T^{1-(0.053\beta+0.0514\beta^2)}}{e^{0.41\beta+0.083\beta^2}} \,. \tag{7}$$

Therefore equation (7) is a 'practical' approach to solutions of equation (5), of particular interest within the 'weak nonlinear range'. Information about the accuracy of T_a to represent $T_{r,a}$, is shown in Figure 3, where the error estimator $\varepsilon = (T_{r,a} - T_a)/\,T_{r,a}$ has been used.

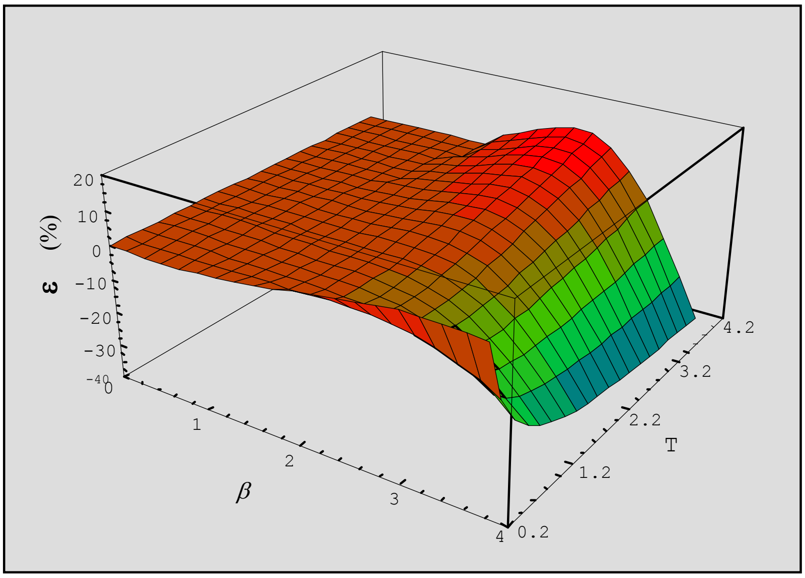

Figure 3 – 'Error' estimation of equation (7).

For $\beta<2$ equation (12) gives nearly 'exact values' for the reverberation time under nonlinear conditions. Even more: up to $\beta<3.5$ errors are bounded to less than 10%. Anycase it isto remember that initial assumptions are only valid for low values of β, say $\beta< 2$.

4 Conclusions

Sound reverberation in rooms under nonlinear conditions has been analysed and compared to reverberation under 'normal' linear conditions.

A nonlinear parameter β representing the 'excess' attenuation of sound intensity with run path length, has been introduced and assumed to be the responsible of changes on reverberation time under nonlinear conditions. Weak nonlinearities of sound intensity are considered (β <2) though analysis has been extended to =4, that clearly corresponds to high nonlinearity.

Reverberation time under nonlinear conditions is lower that under linear conditions. Differences increase as β increases. An explicit equation giving reverberation time as a function of β (and under linear conditions) has been derived.

It has been shown that for signals within the range on weak nonlinearity can lead to noticeably differences on reverberation time. This result is of particular importance in room experiments where some usual nonlinear sound sources are used (pistol or crack reports, for instance).
Abnormally high values of absorption and level variations with distance to source can be obtained.

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