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Study of the effects of signal filtering on the processing gain in acousto-optic correlators

BAZZI Oussama, MOHANNA Yasser, KHALIL Fadi

Physics department, Faculty of Sciences I, Lebanese University, Beirut, LEBANON
obazzi@ul.edu.lb, yamoha@ul.edu.lb, fkhalil@ul.edu.lb

Abstract

The effects of signal filtering on the processing gain in acousto-optic correlators (AOC) in direct-sequence spread-spectrum receivers are considered. Different filters are considered. The results are given in terms of the code rate to the filter bandwidth. Analytical expressions are derived for ideal AOC's. For real AOC's results are obtained from numerical simulation.

Keywords: acousto-optics, correlation, signal-to-noise ratio

1 Introduction

Acousto-optic correlators are considered attractive for processing spread-spectrum (SS) signals as they offer real-time operation and high dynamic range. Different architectures are described in the literature [1-3]. The effects of integrating the AOC in a spread spectrum receiver are presented in this paper.
The AO correlator and the SS receiver are first briefly reviewed. The receiver model is equivalent, in the baseband, to a low-pass filter acting on the received signal. Three different types of filters are considered: ideal low-pass, RC, and Raised-Cosine filters.
The performance of the correlator is measured as the processing gain obtained compared to an ideal system. Analytical derivation is obtained for the three types of filters in the case of ideal AOC's.
For real AOC's the results are obtained from numerical simulation.

2 The AO correlator

The principle of signal correlation is presented in Fig.1. The 2 signals are modulated by ultrasonic waves and are sent in opposing directions in a Bragg cell illuminated by coherent light. The product of the two signals is realized by two successive acousto-optic interactions [1]. The integration is realized by the output lens (L). The incident optical beam I will interact with the first signal s_1 to yield a modulated beam I_1 and aresidual I_0 . In their turn, I_1 and I_0 will , on interaction with s_2, be splitted respectively into I_{11},I_{10} and I_{01} and I_{00} .
The incident beam I is represented by its electric field vector:

$$\mathbf{E_i} = \mathbf{A}e^{j(\omega_0 t - \mathbf{k_0}.\mathbf{r})} \tag{1}$$

Where **A** is the amplitude, ω_0 is the radian frequency, $\mathbf{k_0}$ is the incident wavevector.

t and **r** designate respectively the time and space coordinates.
The acoustic waves of frequency Ω_0 and wavevector **K** are represented by :

$$s_{1,2}(t) = S_{1,2}e^{j(\Omega_0 t \pm \mathbf{K.z})} \tag{2}$$

where $S_{1,2}$=±1 (BPSK modulation). In relation with Fig. 1, one of the signals is time-reversed in order to obtain correlation.
The AO interaction may be considered (at low efficiency) as a linear amplitude modulation associated with frequency and wavevector shifts equal to those of the acoustic waves.
Hence, for an efficiency α, the electric field of the fist diffracted beam is :

$$\mathbf{E_1} = \alpha S_1 \mathbf{A} e^{j(\omega_0+\Omega_0)t - j(\mathbf{k_0}+\mathbf{K}).\mathbf{r}} \tag{3}$$

The residual field is:

$$\mathbf{E_0} = \sqrt{1-\alpha^2}\,\mathbf{E_i} \tag{4}$$

The expression of all other field components are derived in the same manner. Particular interest is in the 2 parallel output beams having close frequencies :

$$\mathbf{E_{00}} = (1-\alpha^2)\mathbf{E_i} \quad \text{and} \quad \mathbf{E_{11}} = S_1(t)S_2(t)e^{2j\Omega_0 t}\alpha^2\mathbf{E_{00}} \tag{5}$$

The detection is realized by the system (Lens + photodiode).
The output current is :

$$\mathbf{i}(t) = \Gamma\cos(2\Omega_0 t)\int_0^L S_1(u)S_2(u-t)du = \Gamma C_{12}(t)\cos(2\Omega_0 t) \tag{6}$$

Where Γ is a constant factor and $C_{12}(t)$ designates the correlation of the two signals S_1 ansd S_2.

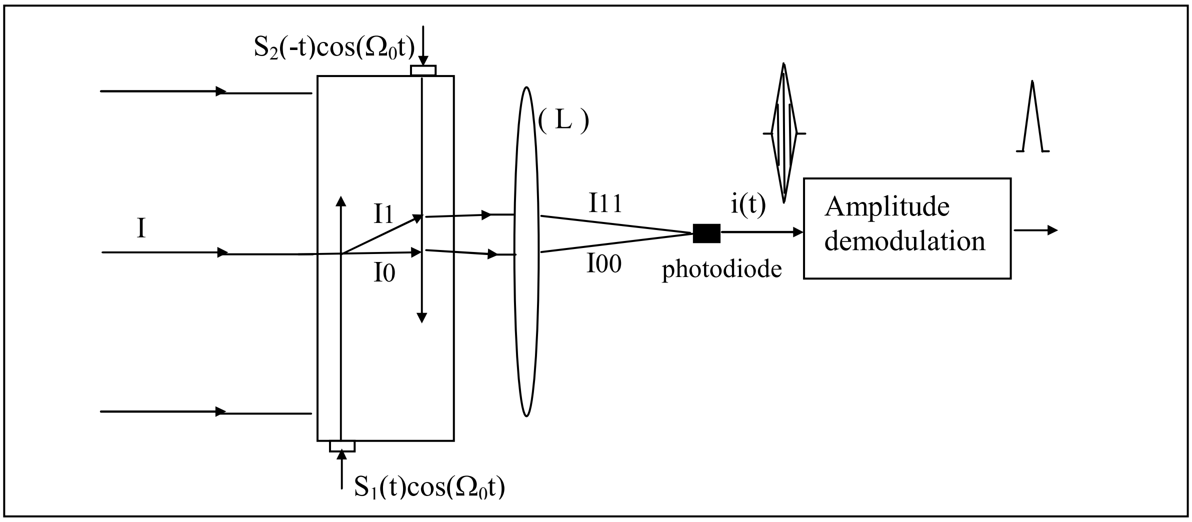

Figure 1 – The AO correlator.

3 The SS receiver

A view of the equivalent SS receiver diagram using an AOC is given in Fig. 1. The receiver role is to down-convert the received SS signal into baseband before performing correlation with a reference code. We distinguish the two filters of frequency responses $H_1(f)$ and $H_2(f)$ where :

1. filter $H_1(f)$ is proper to the AO correlator. It represents the acousto-optic interaction frequency response and affects both the received and the reference signals. The equivalent normalized baseband representation of $H_1(f)$ is [3]:

$$H_1(f) = \mathrm{sinc}(\frac{f}{\Delta f}) \tag{7}$$

Where $\sin c(x) = \frac{\sin(\pi x)}{\pi x}$,

f is the acoustic frequency, and Δf is the cutoff frequency at –4dB corresponding to half the first-null bandwidth.

2. filter $H_2(f)$ is the receiver filter which characterizes the receiver channel. This filter affects the received signal. Three different receiver filters are considered: ideal low-pass, RC filter and raised-cosine filter.

The characteristics of the AO correlator and the different receiver filters are given in Table1.

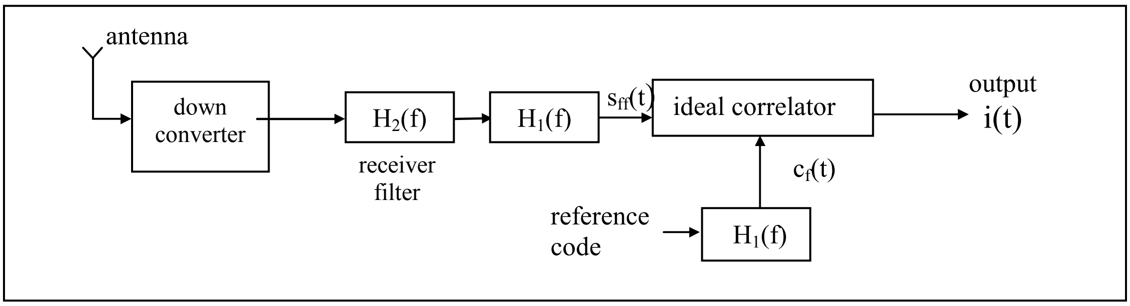

Figure 2 – The SS receiver

4 Analysis of the receiver performance

The received signal s(t) in Fig.1 consists of a desired signal Ac(t) corrupted by an additive zero-mean white Gaussian noise of power spectral density (PSD) equal to η :

$$s(t)=Ac(t)+n(t) \tag{8}$$

where A is the amplitude of the received signal, n(t) denotes a sample function of the noise process.
At the receiver input the signal to noise ratio is:

$$(\mathrm{SNR})_i = \frac{A^2}{\sigma_i^2} = \frac{A^2}{\eta} \tag{9}$$

Where σ_i^2 is the input noise power computed in a bandwidth equal to the chip rate which is, for convenience, normalized to one.
The processing gain is defined as the enhancement obtained in the signal to noise ratio (SNR) as the signal goes through the receiver. In an ideal receiver (infinite bandwidth), the processing gain is equal to the spreading factor N i.e the number of chips in the code period [6]. When N is large, the autocorrelation of the code can be approximated by [6]:

$$C_{cc}(t)=\mathrm{tri}(t) \tag{10}$$

Where the chip rate is normalized to one and tri(t)=1-abs(t) for abs(t)<1 and 0 otherwise.

At the receiver output, referring to Fig. 2, the signal obtained is:

$$i(t)= \frac{1}{N}\,[s_{ff}(t) * c_f^N(-t)] \tag{11}$$

where * is the convolution operator, the reference code is time–reversed in order to obtain correlation. The subscript (ff) refers to a two-stage filtering of signals: first in the receiver filter and second in the AO correlator. The subscript (f) refers to AO filtering alone. The superscript N refers to a single period of the code given by:

$$c^N(t)=c(t)\mathrm{rect}\left(\frac{t}{N}\right) \tag{12}$$

where rect(x)=1 for abs(x)<0.5 and 0 otherwise .

Eq. (11) can be split as :

$$\begin{aligned} i(t) &= \frac{1}{N}[Ac_{ff}(t)+n_{ff}(t)] * c_f^N(-t) \\ &= \frac{A}{N}\,[c_{ff}(t) * c_f^N(-t)] + \frac{1}{N}\,[n_{ff}(t) * c_f^N(-t)] \\ &= s_0(t)+n_0(t) \end{aligned} \tag{13}$$

where $s_0(t)$ and $n_0(t)$ represent respectively signal and noise at the correlator output.
The signal to noise ratio $(SNR)_0$ at the correlator output is defined as the ratio of the signal correlation peak power $\overline{s_0(0)}^2$, at the coincidence instant, to the output noise variance σ_0^2 [7] :

$$(SNR)_0 = \frac{\overline{s_0(0)}^2}{\sigma_0^2} \tag{14}$$

Using Eq.(4) and (9), the SNRE can be written as :

$$\mathrm{SNRE} = \frac{(SNR)_0}{(SNR)_i} = \frac{\eta}{A^2\sigma_0^2}\overline{s_0(0)}^2 \tag{15}$$

4.1 Signal and noise analysis

The detailed analysis of $s_0(t)$ and $n_0(t)$ is found in Ref. [4] and [5] .The expressions for $s_0(t)$ and $C_{n0n0}(t)$ are given by :

$$s_0(t) = A\,[C_{cc}(t)*C_{h1h1}(t)*h_2(t)] \tag{16}$$

and

$$C_{n0n0}(t) = \frac{1}{N}\eta C_{h1h1}(t)*C_{h1h1}(t)*C_{cc}(t)*C_{h2h2}(t) \tag{17}$$

where $h_1(t)$ and $h_2(t)$ are the impulse responses of the AO correlator and the receiver filter respectively. $C_{xx}(t)$ denotes the autocorrelation of x(t).
The output noise variance is given by the autocorrelation peak value $C_{n0n0}(0)$ in Eq. (17).

4.2 Analysis of the correlator performance

The performance is studied in terms of two parameters:
1. the ratio α of the code chip rate (normalized to one) to the cutoff frequency Δf of the AOC:

$$\alpha = \frac{1}{\Delta f} \tag{18}$$

2. the ratio β of the code chip rate to the receiver filter bandwidth.

4.2.1 Analytical results

Analytical results of the SNRE in the system are found in the case of ideal AOC : $H_1(f)=1$.
Eqs. (11) and (12) reduce, in this case, to:

$$s_0(t) = AC_{cc}(t)*h_2(t) \tag{19}$$

and

$$C_{n0n0}(t) = \eta/N\,[C_{cc}(t)*C_{h2h2}(t)] \tag{20}$$

The filter characteristics are given in Table 1.
The SNRE enhancement ratio is obtained from Eqs. 15, 19 and 20 as:

$$\text{SNRE} == \frac{\eta}{A^2 C_{n0n0}(0)}\overline{s_0(0)}^2 \tag{21}$$

The analytical derivations are reported in Ref. [4] and [5]. The results are given in the appendix. The processing gain reduction for the various filters compared to the ideal system is presented in Fig. 3. The figure shows that as β increases, the signal is much more affected than noise by the raised cosine filter. For β=5, an approximate SNRE loss of 5.8 dB is obtained with the raised-cosine filter as compared to respective losses of 1.2 dB and 4.2 dB for RC and ideal low-pass filters.

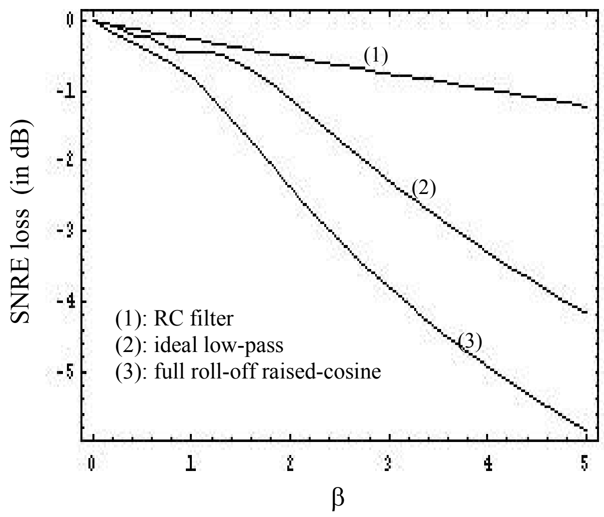

Figure 3 – SNRE loss in the receiver (ideal AOC)

4.2.2 Simulation results

Results in the general case (real AOC) are obtained from numerical simulation of the system (receiver + correlator) as defined in Fig. 2 and as described in Eqs. 16 and 17.
The value N has been taken equal to 1023 with 100 samples per chip.
Results are presented in Fig. 4 for the three filters considered.
We see that as α increases the curves become broader and the AOC filtering effects become dominant.

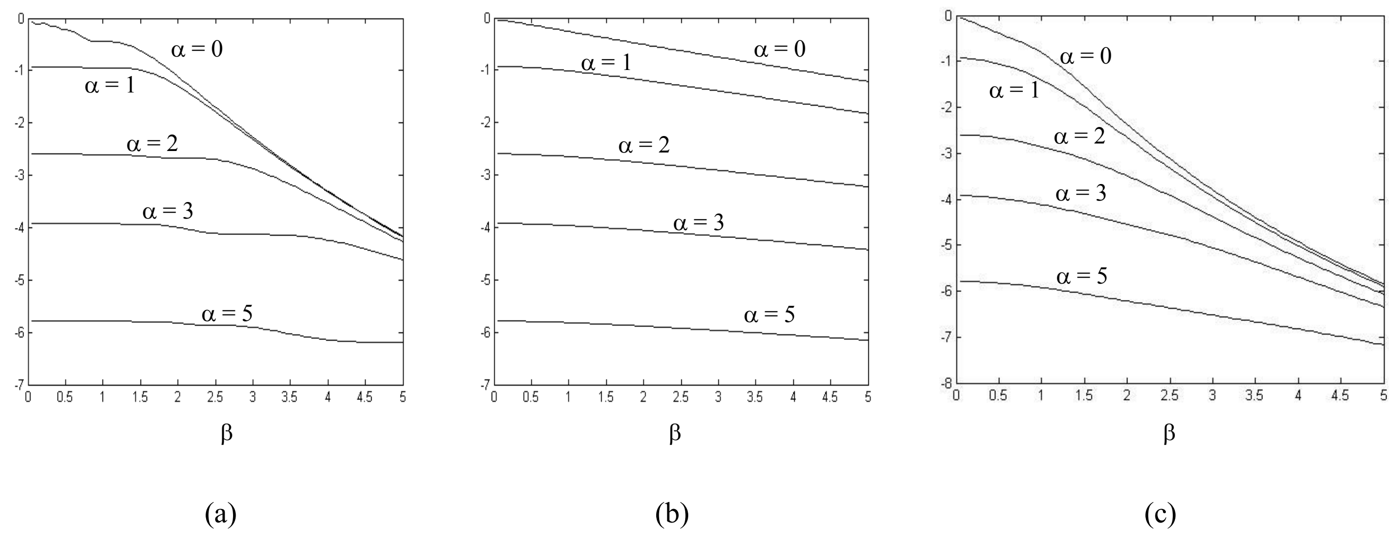

Figure 4 – SNRE losses (in dB) for different receiver filters:
a) Ideal low-pass
b) RC filter
c) full roll-off rasied csosine

5 Conclusion

The effects of different receiver filters on the processing gain in acousto-optic correlators are analyzed using a signal processing approach which is presented. Ideal, RC and full roll-off raised-cosine filters are considered. Analytical expressions are obtained for ideal correlators in terms of the ratio of the code rate to the filter bandwidth. In the general case, results are obtained from numerical simulation.

Table 1 – AO correlator and different receiver filter characteristics.

		Frequency response	*Impulse response*	*Impulse-response autocorrelation*
AO correlator		$H_1(f)=\text{sinc}(\alpha f)$	$h_1(t)=(1/\alpha)\text{rect}(t/\alpha)$	$C_{h1h1}(t)=(1/\alpha)\,\text{tri}(t/\alpha)$
Receiver Filters	**Ideal LPF**	$H_2(f)=\text{rect}(\beta f/2)$	$h_2(t)=2/\beta\,\text{sinc}(2t/\beta)$	$C_{h2h2}(t)=(2/\beta)\text{sinc}(2t/\beta)$
	RC filter	$H_2(f)=\frac{1}{1+jf\beta}$ $(\beta=2\pi RC)$	$h_2(t)=\frac{2\pi}{\beta}e^{\frac{-2\pi}{\beta}t}\quad t\geq 0$ $=0\qquad t<0$	$C_{h2h2}(t)=\frac{\pi}{\beta}e^{-\frac{2\pi}{\beta}\lvert t\rvert}$
	Raised-cosine filter (full roll-off)	$H_2(f)=\frac{1}{2}[1+\cos(\pi\,\beta\,f)]$ $0<\lvert f\rvert<\frac{1}{\beta}$	$h_2(t)=\frac{1}{\beta}\frac{\text{sinc}(x)}{(1-x^2)}$ $(x=\frac{2t}{\beta})$	$C_{h2h2}(t)=$ $\frac{3}{4\beta}\text{sinc}(x)\{1+\frac{5x^2-x^4}{(x^2-1)(x^2-4)}\}$

6 Appendix

The analytical results derived in [4] and [5] for the different filters are here reported.

6.1 Ideal low-pass filter:

$$\mathrm{SNRE} = \mathrm{N}\,\mathrm{I}(\beta) = \mathrm{N}\int_{-1/\beta}^{+1/\beta} \sin c^2(f)\,df \tag{22}$$

6.2 RC filter:

$$\mathrm{SNRE} = \mathrm{N}\frac{\{\frac{1-t_m}{b} + \frac{1}{b^2}(1+e^{-b(t_m+1)} - 2e^{-bt_m})\}^2 b^2}{1+\frac{1}{b}(e^{-b}-1)]} \tag{23}$$

Where b=1/β and $t_m = \frac{\ln(2-e^{-b})}{b} = \frac{\beta \ln(2-e^{-\frac{2\pi}{\beta}})}{2\pi}$ (24)

6.3 Raised-cosine filter:

$$\mathbf{s_0(0)} = \frac{\mathbf{A}}{\mathbf{2\pi}}\left\{\mathbf{2Si(\frac{2\pi}{\beta}) - \beta\, Si(\pi) + (\frac{\beta}{2}+1)\, Si(\pi+\frac{2\pi}{\beta}) + (\frac{\beta}{2}-1) Si\,(\pi-\frac{2\pi}{\beta})}\right\} \tag{25}$$

$$\begin{aligned}\mathbf{C_{n0n0}(0)} = \frac{\eta}{\mathbf{4N\pi}}\{\ &\mathbf{3Si(\frac{2\pi}{\beta}) - \beta\, Si(2\pi) - 2\beta\, Si(\pi) + (\beta+2)\, Si(\pi+\frac{2\pi}{\beta})} + \\ &\mathbf{(\beta-2) Si\,(\pi-\frac{2\pi}{\beta}) + \frac{\beta+1}{2} Si(2\pi+\frac{2\pi}{\beta}) + \frac{\beta-1}{2} Si(2\pi-\frac{2\pi}{\beta})}\ \}\end{aligned} \tag{26}$$

Where Si(x) denotes the Sine-Integral function:

$$\mathrm{Si}(x) = \int_0^x \frac{\sin(t)}{t}dt \tag{27}$$

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